**Q.No.1**

**Ans:**

1. ~ (𝑝 ⋃~𝑞) ⋃ (~𝑝⋂ ~𝑞) ≡~p

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | ~p | ~q | P v ~q | ~p^~q | ~(p v ~q) | ~(p v ~q)v(~P^~q) |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |

Here, ~(p v ~q)v(~P^~q) == ~p. Proved

1. ~ ((~𝑝 ⋂𝑞) ⋃ (~𝑝⋂ ~𝑞)) ⋃(𝑝⋂𝑞) ≡𝑝

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | ~p | ~q | p ^ q | ~p^q | ~p^~q | ((~p^q)v(~p^~q)) | ~((~p^q)v(~p^~q)) | ~((~p^q)v(~p^~q))v(p^q) |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |

Here, ~((~p ^ q)v(~p^~q)) v( p^q) == p. proved

1. (𝑝 ⋂(~(~𝑝⋂𝑞))) ⋃ (𝑝⋂ 𝑞) ≡𝑝

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | ~p | ~p^q | ~(~p^q) | (p^(~(~p^q))) | p^q | (p^(~(~p ^ q))) v (p^q) |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |

Here, (p^(~(~p ^ q))) v (p ^ q)==p. Proved

**Q.No.2**

**Ans: Because an exclusive or statement is false when both components are true and when both components are false.**

**a. (𝑝 ⊕𝑝) ⊕𝑝**

|  |  |  |
| --- | --- | --- |
| **p** | **p** ⊕**p** | **(p** ⊕ **p)** ⊕**p** |
| 1 | 0 | 1 |
| 0 | 0 | 0 |

**b. Is (𝑝 ⊕ 𝑞) ⊕ 𝑟≡𝑝 ⊕ (𝑞 ⊕ 𝑟)?**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **p** ⊕ **q** | **q** ⊕ **r** | **(p**⊕ **q)** ⊕ **r** | **p** ⊕ **(q** ⊕ **r)** |
| 0 | 0 | 0 | FALSE | FALSE | FALSE | FALSE |
| 1 | 0 | 0 | TRUE | FALSE | TRUE | TRUE |
| 0 | 1 | 0 | TRUE | TRUE | TRUE | TRUE |
| 1 | 1 | 0 | FALSE | TRUE | FALSE | FALSE |
| 0 | 0 | 1 | FALSE | TRUE | TRUE | TRUE |
| 1 | 0 | 1 | TRUE | TRUE | FALSE | FALSE |
| 0 | 1 | 1 | TRUE | FALSE | FALSE | FALSE |
| 1 | 1 | 1 | FALSE | FALSE | TRUE | TRUE |

**C. Is (𝑝 ⊕𝑞)⋂𝑟 ≡ (𝑝⋂𝑟) ⊕ (𝑞⋂𝑟) ?**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **p** ⊕ **q** | **(p** ⊕ **q)** ∧ **r** | **p** ∧ **r** | **q** ∧ **r** | **(p** ∧ **r)** ⊕ **(q** ∧ **r)** |
| 0 | 0 | 0 | FALSE | FALSE | FALSE | FALSE | FALSE |
| 1 | 0 | 0 | TRUE | FALSE | FALSE | FALSE | FALSE |
| 0 | 1 | 0 | TRUE | FALSE | FALSE | FALSE | FALSE |
| 1 | 1 | 0 | FALSE | FALSE | FALSE | FALSE | FALSE |
| 0 | 0 | 1 | FALSE | FALSE | FALSE | FALSE | FALSE |
| 1 | 0 | 1 | TRUE | TRUE | TRUE | FALSE | TRUE |
| 0 | 1 | 1 | TRUE | TRUE | FALSE | TRUE | TRUE |
| 1 | 1 | 1 | FALSE | FALSE | TRUE | TRUE | FALSE |

**Q.No.3**

**Ans: Because** p→q is false, p is true and q is false. Hence, ~p is false, and so p→q is true.

1. **~p→q**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p | q | ~p | p→q | ~p→q |
| TRUE | FALSE | FALSE | FALSE | TRUE |

1. **p V q**

|  |  |  |
| --- | --- | --- |
| p | q | p V q |
| TRUE | FALSE | TRUE |

1. **q→p**

|  |  |  |
| --- | --- | --- |
| p | q | q→p |
| TRUE | FALSE | TRUE |

**Q.No.4**

**Ans: A)** *p→(*𝑞*V*𝑟*) ≡ (*𝑝∧*~*𝑞*) →* 𝑟

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | ~q | qvr | p^~q | p-->(qvr) | p^~q-->r | (p-->(qvr))<-->((p^~q)-->r) |
| 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | TRUE |
| 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | TRUE |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | TRUE |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | TRUE |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | TRUE |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | TRUE |
| 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | TRUE |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | TRUE |

Hence, the given statement is tautology.

**B)** *p* ∧ *(*𝑞*V*𝑟*) ≡ (*𝑝∧𝑞*) V (*𝑝∧𝑟*)*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | qvr | p^(qvr) | p^q | p^r | (p^q)v(p^r) | p^(qvr)<-->(p^q)v(p^r) |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | TRUE |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | TRUE |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | TRUE |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | TRUE |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | TRUE |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | TRUE |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | TRUE |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | TRUE |

Hence, the given statement is tautology.

**C)** *p→(*𝑞*→*𝑟*)≡(*𝑝∧𝑞*)→*𝑟

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | q-->r | p-->(q-->r) | p^q | (p^q)-->r | p-->(q-->r)<-->(p^q)-->r |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | TRUE |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | TRUE |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | TRUE |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 | TRUE |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 | TRUE |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 | TRUE |
| 0 | 0 | 1 | 1 | 1 | 0 | 1 | TRUE |
| 0 | 0 | 0 | 1 | 1 | 0 | 1 | TRUE |

Hence, the given statement is tautology.

**Q.No.5**

**Ans:**

1. Suppose the cook was in the kitchen at the time of the murder.

2. The butler killed Lord Hazelton with strychnine.

3. We have a contradiction: Lord Hazelton was killed by strychnine and a blow on the head.

4. The supposition that the cook was in the kitchen is false.

5. The cook was not in the kitchen at the time of the murder.

6. Sara was not in the dining room when the murder was committed.

7. Lady Hazelton was in the dining room when the murder was committed.

8. The chauffeur killed Lord Hazelton.